Package 'GramQuad'

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Title Gram Quadrature

Description Numerical integration with Gram polynomials (based on <arXiv:2106.14875> [math.NA] 28 Jun 2021, by Irfan Muhammad [School of Computer Science, University of Birmingham, UK]).

Depends R (>= 4.0)

Imports pracma, compiler

License GPL-3

URL https://gitlab.com/iagogv/GramQuad

BugReports https://gitlab.com/iagogv/GramQuad/-/issues

Encoding UTF-8

NeedsCompilation no

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Repository CRAN

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`create_gram_weights`   *Compute Gram weights*

Description

Computes weights for Gram quadrature ofm+1 points.

Usage

```
create_gram_weights(m)
```

Arguments

m   a positive integer value, the number of points minus one for which weights will
                 be computed. See 'Details'.

Details

The numerical integration of an analytical function $f(x)$ using a finite set of equidistant points
can be performed by quadrature formulas like the Newton-Cotes. Unlike Gaussian quadrature for-
mulas however, higher-order Newton-Cotes formulas are not stable, limiting the usable order of
such formulas. Existing work showed that by the use of orthogonal polynomials, stable high-order
quadrature formulas with equidistant points can be developed. This algorithm improves upon such
work by making use of (orthogonal) Gram polynomials and deriving an iterative algorithm, together
allowing us to reduce the space-complexity of the original algorithm significantly.

Value

A double-precision vector of the specified length plus one, whose elements are the weights for the
Gram quadratures of the m+1 points in the interval $[-1, 1]$.

Author(s)

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References

Muhammad, Irfan (2021) *Gram quadrature: Numerical integration with Gram polynomials*. arXiv:2106.14875
[math.NA]

See Also

gaussLegendre

Examples

```
m <- 100
xs <- seq(-1, 1, length.out = m + 1)
gram_weights <- create_gram_weights(m)

# the sum of  stable  weights is  equal  to 2.
cat("Sum of Gram weights:", sum(gram_weights), "\n")

# test integration, integrate f below between [-1,1]
f = function(x){ 9 * x ^ 2 + 45 * 13 * x ^ 3 + 16 * x ^ 4}
gram_quad <- sum(gram_weights * f(xs))
cat("Approx. integration:", gram_quad, "\n")
```

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