Package ‘NLPwavelet’

April 4, 2025

Type Package

Title Bayesian Wavelet Analysis Using Non-Local Priors

Version 1.1

Date 2025-04-04

Description Performs Bayesian wavelet analysis using individual non-local priors as described in Sanyal & Ferreira (2017) <DOI:10.1007/s13571-016-0129-3> and non-local prior mixtures as described in Sanyal (2025) <DOI:10.48550/arXiv.2501.18134>.

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Imports Rcpp (>= 1.0.14), wavethresh

LinkingTo Rcpp, RcppArmadillo

URL `https://nilotpalsanyal.github.io/NLPwavelet/`

BugReports `https://github.com/nilotpalsanyal/NLPwavelet/issues`

Repository CRAN

Suggests knitr, rmarkdown

NeedsCompilation yes

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Date/Publication 2025-04-04 18:20:02 UTC

Contents

NLPwavelet-package — *Bayesian Wavelet Analysis Using Non-local Priors*

Description

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Details

The main function is `BNLPWA`, which has arguments for specifying analysis using individual non-local priors or non-local prior mixtures and various hyperparameter specifications for the wavelet coefficients and scale parameters of the non-local priors. See the manual of `BNLPWA` for examples.

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References

Sanyal, Nilotpal. "Nonlocal prior mixture-based Bayesian wavelet regression." arXiv preprint arXiv:2501.18134 (2025).

Sanyal, Nilotpal, and Marco AR Ferreira. "Bayesian wavelet analysis using nonlocal priors with an application to FMRI analysis." Sankhya B 79.2 (2017): 361-388.

BNLPWA — *Bayesian Non-Local Prior-Based Wavelet Analysis*

Description

`BNLPWA` is the main function of this package that performs Bayesian wavelet analysis using individual non-local priors as described in Sanyal & Ferreira (2017) and non-local prior mixtures as described in Sanyal (2025). It currently works with one-dimensional data. The usage is described below.

Usage

```
BNLPWA(
  data,
  func=NULL,
  method=c("mixture","mom","imom"),
  mixprob_dist=c("logit","genlogit","hypsec","gennormal"),
  scale_dist=c("polynom","doubleexp"),
```

```
  r=1,
  nu=1,
  wave.family="DaubLeAsymm",
  filter.number=6,
  bc="periodic"
)
```

Arguments

data	Vector of noisy data.
func	Vector of true functional values. NULL by default. If available, they are used to compute and return mean squared error (MSE) of the estimates.
method	"mixture" for non-local prior mixture-based analysis, "mom" or "imom" for individual non-local prior-based analysis.
mixprob_dist	Specification for the mixture probabilities of the spike-and-slab prior. Equations given in the Details.
scale_dist	Specification for the scale parameters of the non-local priors. Equations given in the Details.
r	Integer specifying a) the order of the MOM prior or the shape parameter of the IMOM prior for individual non-local prior-based analysis, or b) the order of the MOM prior for non-local prior mixture-based analysis.
nu	Integer specifying the shape parameter of the IMOM prior for non-local prior mixture-based analysis. Not used for individual non-local prior-based analysis.
wave.family	The family of wavelets to use - "DaubExPhase" or "DaubLeAsymm". Default is "DaubLeAsymm".
filter.number	The number of vanishing moments of the wavelet. Default is 6.
bc	The boundary condition to use - "periodic" or "symmetric". Default is "periodic".

Details

Spike-and-slab prior for the wavelet coefficients:

For individual MOM prior-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l, \tau_l, \sigma^2, r \sim \gamma_l \ \text{MOM}(\tau_l, \sigma^2, r) + (1-\gamma_l)\ \delta(0),$$

for individual IMOM prior-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l, \tau_l, \sigma^2, r \sim \gamma_l \ \text{IMOM}(\tau_l, \sigma^2, r) + (1-\gamma_l)\ \delta(0),$$

and for non-local prior mixture-based analysis, the spike-and-slab prior for the wavelet coefficient d_{lj} is given by

$$d_{lj} \mid \gamma_l^{(1)}, \gamma_l^{(2)}, \tau_l^{(1)}, \tau_l^{(2)}, \sigma^2, r, \nu \sim \gamma_l^{(1)} \ \text{MOM}(\tau_l^{(1)}, r, \sigma^2) + (1-\gamma_l^{(1)})\gamma_l^{(2)} \ \text{IMOM}(\tau_l^{(2)}, \nu, \sigma^2) + (1-\gamma_l^{(1)})(1-\gamma_l^{(2)})\ \delta(0),$$

where the density of the MOM prior is

$$mom(d_{lj}|\tau_l^{(1)}, r, \sigma^2) = \widetilde{M}_r \left(\tau_l^{(1)}\sigma^2\right)^{-r-1/2} d_{lj}^{2r} \exp\left(-\frac{d_{lj}^2}{2\tau_l^{(1)}\sigma^2}\right), \quad r > 1, \tau_l^{(1)} > 0, \widetilde{M}_r = \frac{(2\pi)^{-1/2}}{(2r-1)!!}$$

and the density of the IMOM prior is

$$imom(d_{lj}|\tau_l^{(2)}, \nu, \sigma^2) = \frac{\left(\tau_l^{(2)}\sigma^2\right)^{\nu/2}}{\Gamma(\nu/2)} |d_{lj}|^{-\nu-1} \exp\left(-\frac{\tau_l^{(2)}\sigma^2}{d_{lj}^2}\right), \quad \nu > 1, \tau_l^{(2)} > 0.$$

Hyperparameter specifications:
For non-local prior mixture-based analysis, the available specifications for the mixture probabilities are

1. **Logit:**

$$\gamma_l^{(1)} = \frac{\exp(\theta_1^\gamma - \theta_2^\gamma l)}{1 + \exp(\theta_1^\gamma - \theta_2^\gamma l)}, \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{\exp(\theta_3^\gamma - \theta_4^\gamma l)}{1 + \exp(\theta_3^\gamma - \theta_4^\gamma l)}, \quad \theta_3^\gamma \in \mathbb{R},\ \theta_4^\gamma > 0$$

2. **Generalized logit or Richards:**

$$\gamma_l^{(1)} = \frac{1}{[1 + \exp\{-(\theta_1^\gamma - \theta_2^\gamma l)\}]^{\theta_3^\gamma}}, \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma, \theta_3^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{1}{[1 + \exp\{-(\theta_4^\gamma - \theta_5^\gamma l)\}]^{\theta_6^\gamma}}, \quad \theta_4^\gamma \in \mathbb{R},\ \theta_5^\gamma, \theta_6^\gamma > 0;$$

3. **Hyperbolic secant:**

$$\gamma_l^{(1)} = \frac{2}{\pi} \arctan\left[\exp\left(\frac{\pi}{2}\left(\theta_1^\gamma - \theta_2^\gamma l\right)\right)\right], \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{2}{\pi} \arctan\left[\exp\left(\frac{\pi}{2}\left(\theta_3^\gamma - \theta_4^\gamma l\right)\right)\right], \quad \theta_3^\gamma \in \mathbb{R},\ \theta_4^\gamma > 0$$

4. **Generalized normal:**

$$\gamma_l^{(1)} = \frac{1}{2} + \text{sign}(\theta_1^\gamma - l)\frac{1}{2\Gamma(1/\theta_2^\gamma)}\gamma\left(1/\theta_2^\gamma, \left|\frac{\theta_1^\gamma - l}{\theta_3^\gamma}\right|^{\theta_2^\gamma}\right), \quad \theta_1^\gamma \in \mathbb{R},\ \theta_2^\gamma, \theta_3^\gamma > 0$$
$$\gamma_l^{(2)} = \frac{1}{2} + \text{sign}(\theta_4^\gamma - l)\frac{1}{2\Gamma(1/\theta_5^\gamma)}\gamma\left(1/\theta_5^\gamma, \left|\frac{\theta_4^\gamma - l}{\theta_6^\gamma}\right|^{\theta_5^\gamma}\right), \quad \theta_4^\gamma \in \mathbb{R},\ \theta_5^\gamma, \theta_6^\gamma > 0$$

For individual non-local prior based analysis, $gamma_l$ is defined likewise.
For non-local prior mixture-based analysis, the available specifications for the scale parameters are

1. **Polynomial decay:**

$$\tau_l^{(1)} = \theta_1^\tau l^{-\theta_2^\tau}, \quad \theta_1^\tau, \theta_2^\tau > 0$$
$$\tau_l^{(2)} = \theta_3^\tau l^{-\theta_4^\tau}, \quad \theta_3^\tau, \theta_4^\tau > 0$$

2. **Double-exponential decay:**

$$\tau_l^{(1)} = \theta_1^\tau \exp(-\theta_2^\tau l) + \theta_3^\tau \exp(-\theta_4^\tau l), \quad \theta_1^\tau, \theta_2^\tau, \theta_3^\tau, \theta_4^\tau > 0$$
$$\tau_l^{(2)} = \theta_5^\tau \exp(-\theta_6^\tau l) + \theta_7^\tau \exp(-\theta_8^\tau l), \quad \theta_5^\tau, \theta_6^\tau, \theta_7^\tau, \theta_8^\tau > 0$$

For individual non-local prior based analysis, tau_l is defined likewise.

Note: The wavelet computations are performed by using the R package **wavethresh**.

Value

A list containing the following.

`data`	The data vector.
`func.post.mean`	Posterior estimate (mean) of the function that generated the data.
`wavelet.empirical`	Empirical wavelet coefficients obtained by wavelet transformation of the data.
`wavelet.post.mean`	Posterior estimate (mean) of the true wavelet coefficients obtained by wavelet transformation of the underlying function.
`hyperparam`	Estimates of the hyperparameters that specify the spike-and-slab prior for the wavelet coefficients.
`sigma`	Estimate of the standard deviation of the error.
`MSE.mean`	Mean squared error of the estimates, computable only if true functional values are supplied in the input argument `func`.
`runtime`	System run-time of the function.

Author(s)

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References

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See Also

`wd`, `wr`

Examples

```
# Using the well-known Doppler function to
# illustrate the use of the function BNLPWA

# set seed for reproducibility
set.seed(1)

# Define the doppler function
doppler <- function(x) {
  sqrt(x * (1 - x)) * sin((2 * pi * 1.05) / (x + 0.05))
}

# Generate true values over a grid of length an integer power of 2
n <- 128  # Number of points
```

```
x <- seq(0, 1, length.out = n)
true_signal <- doppler(x)

# Add noise to generate data
sigma <- 0.2  # Noise level
y <- true_signal + rnorm(n, mean = 0, sd = sigma)

# BNLPWA analysis based on MOM prior using logit specification
# for the mixture probabilities and polynomial decay
# specification for the scale parameter
fit_mom <- BNLPWA(data=y, func=true_signal, r=1, wave.family=
  "DaubLeAsymm", filter.number=6, bc="periodic", method="mom",
  mixprob_dist="logit", scale_dist="polynom")

plot(y,type="l",col="grey") # plot of data
lines(fit_mom$func.post.mean,col="blue") # plot of posterior
# smoothed estimates
fit_mom$MSE.mean


  # BNLPWA analysis using non-local prior mixtures using generalized
  # logit (Richard's) specification for the mixture probabilities and
  # double exponential decay specification for the scale parameter
  fit_mixture <- BNLPWA(data=y, func=true_signal, r=1, nu=1, wave.family=
    "DaubLeAsymm", filter.number=6, bc="periodic", method="mixture",
    mixprob_dist="genlogit", scale_dist="doubleexp")

  plot(y,type="l",col="grey") # plot of data
  lines(fit_mixture$func.post.mean,col="blue") # plot of posterior
  # smoothed estimates
  fit_mixture$MSE.mean


# Compare with other wavelet methods
library(wavethresh)
wd <- wd(y, family="DaubLeAsymm", filter.number=6, bc="periodic") # Wavelet decomposition

wd_thresh_universal <- threshold(wd, policy="universal", type="hard")
fit_universal <- wr(wd_thresh_universal)
MSE_universal <- mean((true_signal-fit_universal)^2)
MSE_universal

wd_thresh_sure <- threshold(wd, policy="sure", type="soft")
fit_sure <- wr(wd_thresh_sure)
MSE_sure <- mean((true_signal-fit_sure)^2)
MSE_sure

wd_thresh_BayesThresh <- threshold(wd, policy="BayesThresh", type="hard")
fit_BayesThresh <- wr(wd_thresh_BayesThresh)
MSE_BayesThresh <- mean((true_signal-fit_BayesThresh)^2)
MSE_BayesThresh

wd_thresh_cv <- threshold(wd, policy="cv", type="hard")
```

```
fit_cv <- wr(wd_thresh_cv)
MSE_cv <- mean((true_signal-fit_cv)^2)
MSE_cv

wd_thresh_fdr <- threshold(wd, policy="fdr", type="hard")
fit_fdr <- wr(wd_thresh_fdr)
MSE_fdr <- mean((true_signal-fit_fdr)^2)
MSE_fdr

# Compare with non-wavelet methods
    # Kernel smoothing
fit_ksmooth <- ksmooth(x, y, kernel="normal", bandwidth=0.05)
MSE_ksmooth <- mean((true_signal-fit_ksmooth$y)^2)
MSE_ksmooth
    # LOESS smoothing
fit_loess <- loess(y ~ x, span=0.1)  # Adjust span for more or less smoothing
MSE_loess <- mean((true_signal-predict(fit_loess))^2)
MSE_loess
    # Cubic spline smoothing
spline_fit <- smooth.spline(x, y, spar=0.5)  # Adjust spar for smoothness
MSE_spline <- mean((true_signal-spline_fit$y)^2)
MSE_spline
```

Index